Electrostatics & Magnetostatics

1. Introduction

The theory which describes physical phenomena related to the interaction between stationary electric charges or charge distributions in space with stationary boundaries is called electrostatics. For a long time electrostatics, under the name electricity was considered an independent physical theory of its own, alongside other physical theories such as magnetism, mechanics, optics and thermodynamics.

1.1. Coulomb's law

It has been found experimentally that in classical electrostatics the interaction between stationary, electrically charged bodies can be described in terms of a mechanical force. Let F denote the force acting on a electrically charged particle with charge q located at x, due to the presence of a charge q´ located at x´. According to Coulomb's law this force is, in vacuum, given by the expression

$$F(x) = \frac{qq'}{4\pi\varepsilon_0}\frac{x-x'}{|x-x'|^3} \qquad (1)$$

In SI units, the force F is measured in Newton (N), the electric charges q and q´ in Coulomb (C) and the length $|x - x'|$ in metres (m). The constant ε_0 = 8:8542 $\times 10^{-12}$ Farad per metre (F/m).

1.2. The Electrostatic Field

Instead of describing the electrostatic interaction in terms of a 'force action at a distance', it turns out that it is for most purposes more useful to introduce the concept of a field and to describe the electrostatic interaction in terms of a static vectorial electric field E_{stat} defined by the limiting process

$$E_{stat} = \lim_{q\to 0}\frac{F}{q} \qquad (2)$$

where F is the electrostatic force, from a net electric charge q´ on the test particle with a small electric net electric charge q.

1.3. Magnetostatics

While electrostatics deals with static electric charges, magnetostatics deals with stationary electric currents, i.e., electric charges moving with constant speeds, and the interaction between these currents. Here we shall discuss this theory in some detail.

1.3.1 Ampère's law

Experiments on the interaction between two small loops of electric current have shown that they interact via a mechanical force, much the same way that electric charges interact.

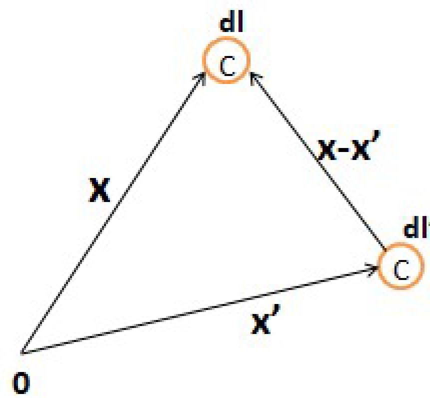

Figure 1.: Ampère's law describes how a small loop C, carrying a static electric current I through its tangential line element dl located at x, experiences a magnetostatic force from a small loop C´, carrying a static electric current I´ through the tangential line element dl´ located at x´. The loops can have arbitrary shapes as long as they are simple and closed.

In Figure 1, let F denote such a force acting on a small loop C, with tangential line element dl, located at x and carrying a current I in the direction of dl, due to the presence of a small loop C´, with tangential line element dl´, located at x´ and carrying a current I´ in the direction of dl´. According to Ampère's law this force is, in vacuum, given by the expression

$$F\,(x) = \frac{\mu_0 I I'}{4\pi} \oint dl \times \oint dl' \times \frac{(x - x')}{|x - x'|^3} \qquad (3)$$

In SI units, $\mu_0 = 4\pi \times 10^{-7}$ H/m is the vacuum permeability. From the definition of ε_0 and μ_0 (in SI units) we observe that

$$\varepsilon_0 \mu_0 = \frac{10^7}{4\pi c^2}(\text{F/m}) \times 4\pi \times 10^{-7}\,(\text{H/m}) = \frac{1}{4\pi c^2}\,(\text{s}^2/\text{m}^2)$$

which is a most useful relation.

1.3.2 The magnetostatic field

In analogy with the electrostatic case, we may attribute the magnetostatic interaction to a static vectorial magnetic field B_{stat}. It turns out that the elemental B_{stat} can be defined as $dB_{stat}\,(x)$

$$dB_{stat}\,(x) = \frac{\mu_0 I'}{4\pi} dl' \times \frac{(x - x')}{|x - x'|^3} \qquad (4)$$

which expresses the small element $dB_{stat}(x)$ of the static magnetic field set up at the field point x by a small line element dl´ of stationary current I´ at the source point x´. The SI unit for the magnetic field, sometimes called the magnetic flux density or magnetic induction, is Tesla (T). If we generalize expression (4) to an integrated steady state electric current density j(x), measured in A/m^2 in SI units, we obtain Biot-Savart's law:

$$B_{stat}(x) = \frac{\mu_0}{4\pi}\int d^3x'\, j(x') \times \frac{(x-x')}{|x-x'|^3} \qquad (5)$$

1.4. Electromagnetic Potentials

Instead of expressing the laws of electrodynamics in terms of electric and magnetic fields, it turns out that it is often more convenient to express the theory in terms of potentials. This is particularly true for problems related to radiation. Now we will introduce and study the properties of such potentials and shall find that they exhibit some remarkable properties which elucidate the fundamental aspects of electromagnetism and lead naturally to the special theory of relativity.

1.4.1. The electrostatic scalar potential

The electrostatic field $E_{stat}(x)$ is irrotational. Hence, it may be expressed in terms of the gradient of a scalar field. If we denote this scalar field by $-\phi_{stat}(x)$, we get

$$E_{stat}(x) = -\nabla\phi_{stat}(x) \qquad (6)$$

Taking the divergence of this and using Equation (6), we obtain Poisson's equation.

$$\nabla^2\phi_{stat}(x) = -\nabla . E_{stat}(x) = -\frac{\rho(x)}{\varepsilon_0} \qquad (7)$$

A comparison with the definition of E_{stat}, namely Equation (5), shows that this equation has the solution

$$\phi_{stat}(x) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(x')}{|x-x'|} d^3x' + \alpha \qquad (8)$$

where the integration is taken over all source points x′ at which the charge density $\rho(\mathbf{x}')$ is non-zero and α is an arbitrary quantity which has a vanishing gradient. An example of such a quantity is a scalar constant. The scalar function $\phi_{stat}(x)$ in Equation (8) is called the electrostatic scalar potential.

1.4.2. The magnetostatic vector potential

Consider the equations of magnetostatics, we know that any 3D vector ***a*** has the property that $\nabla.(\nabla \times \mathbf{a}) = 0$ and in the derivation of Equation in magnetostatics we found that $\nabla. \mathbf{B}_{stat}(\mathbf{x}) = 0$. We therefore realize that we can always write

$B_{stat}(\mathbf{x}) = \nabla \times \mathbf{A}_{stat}(\mathbf{x})$

where $\mathbf{A}_{stat}(\mathbf{x})$ is called the magnetostatic vector potential.

We saw above that the electrostatic potential (as any scalar potential) is not unique: we may, without changing the physics, add to it a quantity whose spatial gradient vanishes. A similar arbitrariness is true also for the magnetostatic vector potential.